Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Advanced Complex Analysis

Subject Code : 5SC04CAC1 **Branch: M.Sc (mathematics)**

Semester : IV **Date 05/05/2016** **Time : 2:30 pm To 5:30 pm** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions .** **(07)**

a. What is path? **(02)**

b. Define function of bounded variarion. **(02)**

c. Evaluate $\oint_C \frac{z}{z^2-1}$ where C: $|z|$=3 **(02)**

d. Define closed rectifiable curve. **(01)**

Q-2 **Attempt all questions** **(14)**

a. Let γ:[0,1]→ $\mathbb{C}$ be closed rectifiable curve and a $\notin$ $\{\gamma\}$then show that $\frac{1}{2\pi i}\int_\gamma \frac{dz}{z-a}$ is an integer. **(07)**

b. Let γ be a closed rectifiable curve in $\mathbb{C}$ then show that n(γ ; a) is constant on every component of $\mathbb{C}$~{ γ } . **(07)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Cauchy's integral formula for derivative . **(07)**

b. State and prove Morera's theorem. **(07)**

Q-3 **Attempt all questions** **(14)**

a. State and prove counting zero principle. **(07)**

b. Evaluate $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}$dz Where f(z)=$(z^2+1)^3$ and γ(t) = $2e^{it}$, 0≤ t ≤ 2π . **(04)**

c. Evaluate $\frac{1}{2\pi i}\int_{|z|=2}\frac{2z+1}{z^2+z+1}\,dz$ **(03)**

OR

Q-3 **a.** Let f be an entire function show that infinity is a pole of order m of f if and only if f is a polynomial of degree m. **(07)**

b. State and prove argument principle. **(05)**

c. Write the statement of open mapping theorem. **(02)**

SECTION – II

Q-4 **Attempt the Following questions .** **(07)**

a. Define convex set. **(02)**

b. Write the statement of Casorati-Weierstrass theorem. **(02)**

c. What is conformal map ? **(02)**

d. Define equicontinous set in C(G, Ω). **(01)**

Q-5 **Attempt all questions** **(14)**

a. Prove that a map f:[a,b]→R is convex iff $\frac{f(t)-f(s)}{t-s} \leq \frac{f(u)-f(t)}{u-t}$ whenever s,t,u ∈ [a,b] and $s < t < u$. **(07)**

b. State and prove Arzela –Ascoli theorem. **(04)**

c. State Hadamard's three circles theorem . **(03)**

OR

Q-5 State and prove Montel's theorem **(14)**

Q-6 **Attempt all questions** **(14)**

a. Prove that If $|a| <1$, then φ_a is one –one map of D onto itself and the inverse of φ_a is φ_{-a} further φ_a maps the boundary of D onto itself. **(07)**

b. State and prove Riemann mapping theorem. **(07)**

OR

Q-6 **Attempt all Questions**

a. If $|z| < \frac{1}{2}$, then show that $\frac{1}{2}|z| < |\log(1+z)| < \frac{3}{2}|z|$ **(07)**

b. Let z_n be sequence of complex number with Re $z_n > 0$ for all then show that the product $\prod_{n=1}^{\infty} z_n$ converges to a non zero complex number iff the series $\sum_{n=1}^{\infty} \log(z_n)$ converges. **(07)**